Electricity and Magnetism, Exam 1, 18/02/2021

3 questions, 55 points

Write your name and student number on each answer sheet. Use of a calculator is allowed. You may make use of the book and the provided formula sheet. The same notation is used as in the book, i.e. a bold-face $\mathbf{A}$ is a vector, $\hat{x}$ is the unit vector in the x-direction, and T is a scalar. **In your handwritten answers, remember to indicate vectors (unit vectors) with an arrow (hat) above the symbol**.

Submit a pdf of the handwritten sheet of paper with your answers (**use a separate sheet for each question!**) to the corresponding assignment in the Nestor Exam environment.

1. Mathematical tools and techniques

   (a) (5 points) Calculate the divergence of $\mathbf{v} = (\cos\theta/r^2)\hat{r} + (r/\sin\theta)\hat{\theta} + (r\cos\theta\sin\phi)\hat{\phi}$

   (b) (5 points) Calculate the curl of $\mathbf{v} = s(2 + \sin^2\phi)\hat{s} + s\sin\phi\cos\phi\hat{\phi} + 3sz\phi\hat{z}$

   (c) (5 points) Give an example of a two-dimensional vector field (in the x,y plane) with **positive divergence and a non-zero curl**. Provide the formula of the vector field, explicitly calculate the divergence and curl. Bonus question (+2) if you have time left: make a sketch of the field.

   (d) (5 points) Find the volume and total surface area of a thick spherical shell with inner radius a and outer radius $2a$, through integration.

2. Circles and spheres

   (a) (5 points) Three small positive charges $(+q, +2q, +3q)$ are enclosed by three closed surfaces (S_1, S_2, S_3), as shown in the figure. The net electric flux through S_1 is Φ_E. What is the net electric flux through S_2, in units of Φ_E?
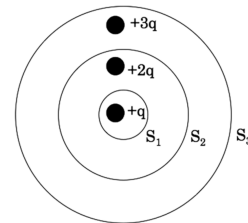

   (b) (5 points) Calculate the divergence of $\mathbf{v} = \frac{1}{2r^2}\hat{\mathbf{r}}$.

   (c) (10 points) Find the electric field at an arbitrary point P on the central axis between two circular loops. The upper loop has radius r, the lower loop radius $r/2$, and their distance is z. The top (large) loop carries a uniform line charge q per unit length, the bottom (small) loop carries a total charge of $2q$.
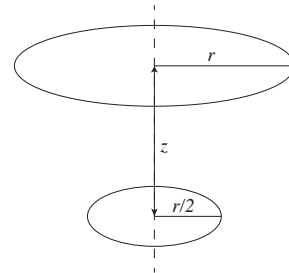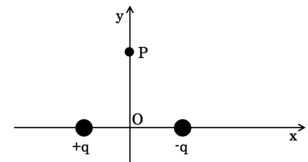
3. Charges, cubes and lines

   (a) (5 points) Four charges are positioned on the corners of a cube with size d, as in the figure on the right. The two charges next to each other on one side of the cube each have charge $+q$, the two others each have charge $-q$. What is the electric field (magnitude and direction) in the center of the cube?
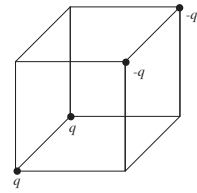
   (b) (5 points) Two charges q sit at the neighbouring corners of a cube with size d, as in the figure on the right. What is the flux of **E** through the shaded side? Explain how you found your answer.
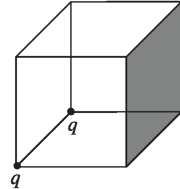
   (c) (5 points) Two charges $+q$ and $-q$ are located along the x-axis at points that are equidistant from the origin, as shown in the figure. Point P and the two charges form an equilateral triangle (all sides the same length a). What is the electric field direction and magnitude at point P on the y-axis?

The End